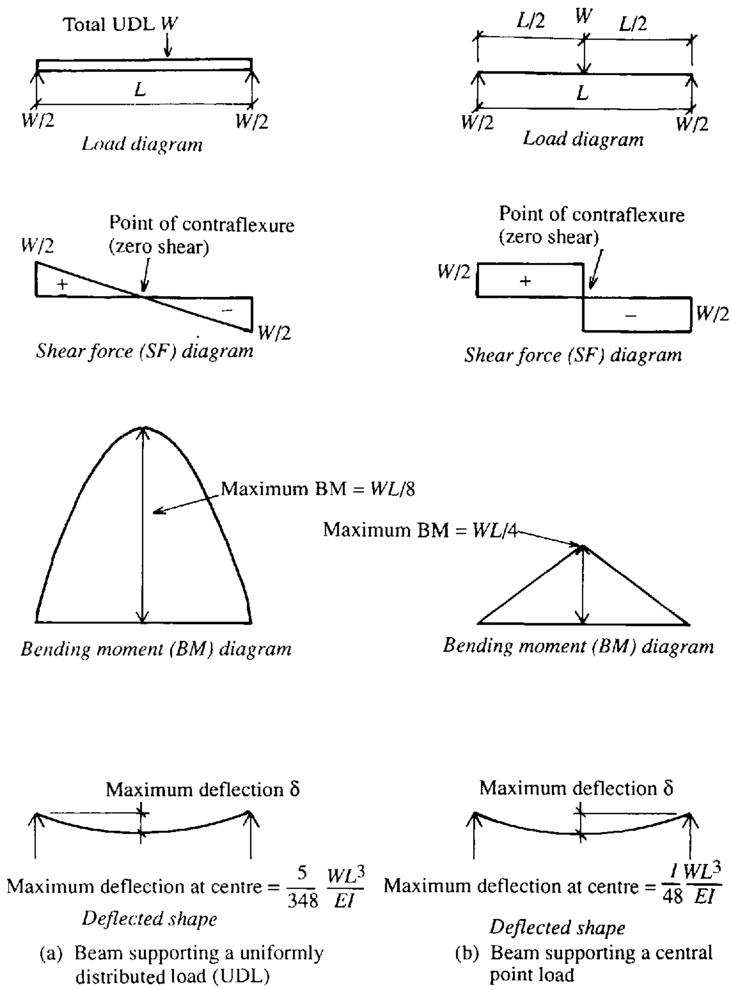

Figure 1.14 *Load, shear force and bending moment diagrams for standard loading conditions*

The resistance of a beam to bending, referred to in item (c), is derived from the theory of bending. The general expression for the theory of bending is

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

where

M either the internal moment of resistance (MR) of the beam or the external bending moment (BM) applied to the beam

I second moment of area of the beam which is a geometrical property of the beam

f stress value for the beam (dependent on the beam material, such as timber or steel)

y distance from the neutral axis (NA) of the beam to its extreme fibres

E Young's modulus of elasticity for the beam (again dependent on the beam material)

R radius of curvature after bending

The term E/R relates to the deformation of a beam and is used in the derivation of deflection formulae. It is not used in bending calculations, and the expression therefore reduces to

$$\frac{M}{I}=\frac{f}{y}$$

This expression may be rearranged so that

$$M=f\frac{I}{y} \quad \text{or} \quad f=M\frac{y}{I}$$

Now I/y is a geometric property of a beam section called the elastic modulus or section modulus, and is denoted by the symbol Z. Thus

$$M=fZ \tag{1.1}$$

or

$$f=\frac{M}{Z} \tag{1.2}$$

or

$$Z=\frac{M}{f} \tag{1.3}$$

The equations can be used in design as follows:

(a) Equation 1.1 may be used to calculate the internal moment of resistance (MR) for a beam of known size (Z known) and material (f known).

(b) Equation 1.2 may be used to calculate the bending stress f occurring within a beam of known size (Z known) when it is subjected to an externally applied bending moment (BM known).

(c) Equation 1.3 may be used for a beam of known material (f known) to calculate the beam property Z needed for the beam to resist an externally applied bending moment (BM known).

The key to their use is the relationship between a beam's moment of resistance (MR) and the applied bending moment (BM). If a beam section is not to fail under load, an internal moment of resistance (MR) must be developed within the beam at least equal to the maximum external bending moment (BM) produced by the loads. That is,

$$\text{Internal MR} = \text{external BM}$$